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Supersymmetry and chiral symmetry

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Abstract: We dispute the nonperturbative nonrenormalisation theorem stating that mass cannot be spontaneously generated in supersymmetric QED. Our analysis, which requires no truncation and is fully gauge and supersymmetry consistent, finds instead that there is no reason for the mass corrections to be exactly zero. We concede that an achiral solution is yet to be found. We also extend a long-standing perturbative result, that the effective potential is zero to all orders of perturbation theory, to the nonperturbative regime for arbitrary numbers of flavours.

Keywords: Nonperturbative Effects, Spontaneous Symmetry Breaking, Supersymmetry Phenomenology.

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Contents

1. Introduction

The supersymmetric (SUSY) nonrenormalisation theorem for mass to all orders in perturbation theory has been established for some time [1, 2, 3]. It has been claimed [4, 5, 6] that a stronger form of this theorem follows from a nonperturbative analysis in SUSY quantum electrodynamics (SQED). This theorem asserts that the chiral solution to the Dyson-Schwinger equation (DSE) in SQED is not merely favoured but unique, i.e. that there is no achiral solution. The claim was first made by Clark and Love [4] who, using the superfield formalism and truncating diagrams containing seagull and higher order n-point vertices, found that the effective mass $\mathcal{M}$ contains a prefactor $\frac{1}{\xi} - 1$ which vanishes in Feynman gauge. Their principal argument is that if the mass vanishes in one gauge then it must vanish in all gauges. They therefore conclude that there can be no dynamic chiral symmetry breaking in SQED, even beyond the rainbow approximation. Their result is comparable to that of a paper by Buchmuller and Love [7] in which the SUSY NJL model is also seen to have no achiral solution to its DSE.

This approach was criticized by Kaiser and Selipsky on two grounds [5]. Firstly they argue that the truncation of seagull diagrams is too severe as it ignores contributions even at the one-loop level. Secondly they suggest that infinities arising from the infrared divergences that plague the superfield formalism can counter the vanishing prefactor and allow spontaneous mass generation. They also point out that the original nonrenormalisation theorem did not forbid mass corrections, only infinite mass counter-terms.

The issue was taken up by Campbell-Smith and Mavromatos [6] who investigated chiral symmetry breaking in $2+1$ dimensional SQED ($SQED_3$) using superfields with both two- and four-component spinors. In the four-component theory [6] they also find a nonrenormalisation theorem. Their analysis dimensionally reduces $SQED_4$ to $SQED_3$, introducing a compactification scale in the process. After truncating all two-particle irreducible diagrams from the DSE, taking the limit that all momenta are small compared to the momentum scale of the compactification, and making several other approximations, they find the same prefactor in front of the effective mass as Clark and Love, and claim that its cancellation by infrared divergences is subverted by the lack of a corresponding prefactor in the renormalisation factor $\mathcal{Z}$. Since their argument depends on dimensional reduction of $SQED_4$, it cannot be applied in $3+1$ dimensions.

The authors of ref. [4] support their argument with a calculation in the bare approximation in Landau gauge ($\xi \to \infty$), which relied on the wavefunction renormalisation being uniformly equal to one, as believed by many authors of the time in ordinary QED [8]–[12]. However this is now considered to be false [13, 14, 15], and so the calculation, in hindsight, is unreliable.

Koopmans and Steringa's analysis [16] found that chiral symmetry was broken for flavour number less than 3.4 flavours with two-component fermions. Their analysis was one of the earliest attempts to go beyond the bare approximation in SQED. They dress the vertices by multiplying them by the wave-function renormalisation. Indeed, the vertices used in this paper reduce to those in their analysis in the limit of zero momentum transfer if one assumes that the wave-function and mass renormalisation are constant functions of the electron/selectron momentum, i.e. contributions from the derivatives of the renormalisation functions are neglected. This is a questionable approach as these functions vary considerably in the low momentum limit where the nonperturbative dynamics are largely determined. The question of chiral symmetry breaking in SQED3 with four-component fermions was addressed by Pisarski who argued that chiral symmetry remains unbroken in the limit $N \to \infty$, where N is the number of flavours. However neither of these papers denied the existence of an achiral solution to the DSE, only that the chiral solution was dynamically preferred.

Our analysis in the component formalism finds no evidence for a nonrenormalisation theorem in SQED. It is certainly the case that no vanishing gauge dependent prefactor emerges in our analysis. We do not dispute the prefactor's existence in the superfield formalism, or the validity of Clark and Love's equations within the limitations of their approximation. However we agree with Kaiser and Selipsky in disputing the validity of their gauge dependent argument and suggest that the rebuttal to Kaiser and Selipsky offered by Campbell-Smith and Mavromatos is a consequence of the extensive truncations used in that paper. By using the component form of the DSE, we free ourselves from having truncations of any kind. Indeed, we perform our calculations using the most general form of the vertices possible within the constraints of SUSY and U(1) gauge invariance [17].

The CJT effective potential [18] expresses the potential of a theory as a functional of its propagators and can be used to compare the chiral and achiral propagators in ordinary field theories. It has been known for some time that in perturbation theory [19, 20] the

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effective potential is exactly zero to all orders in a SUSY theory, and Pisarski has adapted these proofs to the many flavour limit in the nonperturbative theory [21]. However, a rigorous result in perturbation theory cannot be assumed to hold in the nonperturbative regime. So, while the favoured solution must have a potential of exactly zero since SUSY is unbroken [22], it is still reasonable to ask if the unfavoured one does not.

There has been some confusion regarding the calculation of the CJT effective potential (*eg.* see [23]) in SUSY theories. This arises from the fact that it is not clear how to treat Green's functions with auxiliary fields. This problem was solved in a previous paper [17], and substituting the propagators from that paper into the CJT effective potential yields that it is uniformly zero. This renders the CJT effective potential ineffective for choosing between solutions to the DSE in the absence of SUSY breaking.

Our notation and an outline of the derivation of the SUSY Ward identities (SWIs) is given in section 2. We also show that the expected complications of the nonlinearity of the SUSY transforms are avoided because the abelian nature of SQED decouples the ghost superfields and allows us to avoid the complications of a full BRST derivation. Section 6 explains, with extreme brevity, the derivation of the nonrenormalisation theorem in the superfield notation. Section 7 gives our argument against the existence of a nonperturbative nonrenormalisation theorem. Substituting in the most general possible form for the three-point vertices [17] (within certain highly reasonable assumptions), we find that it is impossible to find an acceptable *ansatz* in which the effective mass must be zero. However we do not succeed in finding an achiral solution so the (remote) possibility that the DSE cannot be solved with a dressed mass remains open.

Our analysis of the CJT effective potential, in which we show that it is uniformly zero, is presented in section 8. This result is not spoiled by vacuum polarisation.

2. Analysis in the Component Notation

The full lagrangian of SQED has long been known to be an exponential expansion [24] and it has generally been common practice to take the Wess-Zumino (super)gauge. This reduces the lagrangian to the manageable polynomial form

$$\begin{aligned} L = {}& |f|^2 + |g|^2 + |\partial_\mu a|^2 + |\partial_\mu b|^2 - \bar{\psi}\, \not{\partial}\psi - m(a^* f + a f^* + b^* g + b g^* + i\bar{\psi}\psi) - \\ & -ieA^\mu (a^* \overleftrightarrow{\partial}_\mu a + b^* \overleftrightarrow{\partial}_\mu b + \bar{\psi}\gamma_\mu\psi) - e[\bar{\lambda}(a^* + i\gamma_5 b^*)\psi - \bar{\psi}(a + i\gamma_5 b)\lambda] + \\ & +ieD(a^* b - a b^*) + e^2 A_\mu A^\mu (|a|^2 + |b|^2) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\bar{\lambda}\, \not{\partial}\lambda + \frac{1}{2}D^2 \,. \end{aligned} \tag{2.1}$$

This is invariant to U(1) gauge transformations but the use of the Wess-Zumino (WZ) gauge means that it is not invariant to pure SUSY transformations but to SUSY transformations followed by a special supergauge transformation [24]. The modified SUSY transformation that leaves SQED invariant is given by

$$\begin{aligned} \delta_{WZ} a &= -i\bar{\zeta}\psi \,, \\ \delta_{WZ} b &= \bar{\zeta}\gamma_5\psi \,, \\ \delta_{WZ} \psi &= [f + i\gamma_5 g + i\gamma\cdot\partial(a + i\gamma_5 b) - e\gamma\cdot A(a - i\gamma_5 b)]\zeta \,, \end{aligned}$$

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$$
\begin{aligned}
\delta_{WZ} f &= \bar{\zeta}[\gamma \cdot \partial\psi + e[-a\lambda - ib\gamma_5\lambda + i\gamma \cdot A\psi]]\,, \\
\delta_{WZ} g &= i\bar{\zeta}[\gamma_5\gamma \cdot \partial\psi + e[-\gamma_5 a\lambda - ib\lambda - i\gamma \cdot A\gamma_5\psi]]\,, \\
\delta_{WZ} A_\mu &= \bar{\zeta}\gamma_\mu\lambda\,, \\
\delta_{WZ}\lambda &= \sigma^{\nu\mu}\partial_\mu A_\nu\zeta + i\gamma_5 D\zeta\,, \\
\delta_{WZ} D &= i\bar{\zeta}\gamma_5\gamma \cdot \partial\lambda\,,
\end{aligned} \tag{2.2}
$$

The photon and photino fields are A_μ and λ, respectively. The electron is represented by ψ. It makes up the chiral multiplet together with the selectrons a, f, b, g.

Since we also wish to compare with the $2+1$ dimensional results of Campbell-Smith and Mavromatos [6], we shall denote the dimension of the theory by d, where d can be either 3 or 4. Note that in SQED3 there is an additional scalar field K corresponding to the component of the vector field A_μ lost by dimensional reduction. It's vertices are the dimensionally reduced form of the photon vertices and its presence is henceforth implicit when $d = 3$.

The DSE for the electron in SQED is given by [17]

$$
\begin{aligned}
S^{-1}(p) - S_0^{-1}(p) = -\int \frac{d^d q}{(2\pi)^4} \Big\{ & D_{\mu\nu}(p-q)\gamma^\mu S(q)\Gamma^\nu_{\bar{\psi}A_\mu\psi}(q,p) + \\
& + S_\lambda(p-q) D_{aa}(q)\Gamma_{\bar{\lambda}a^*\psi}(q,p) + \\
& + S_\lambda(p-q) D_{af}(q)\Gamma_{\bar{\lambda}f^*\psi}(q,p) \Big\}\,,
\end{aligned} \tag{2.3}
$$

S_0 denotes the bare propagator and the photon and photino propagators are denoted by $D_{\mu\nu}(p-q)$ and $S_\lambda(p-q)$ respectively. This is clearly written in WZ gauge since with full supergauge invariance there are an infinite number of terms arising from the exponential form of the lagrangian.

To derive useful results from this equation it is necessary to insert a suitable expression for the photon and photino vertices. These vertices are governed by their own DSEs which ideally would be solved in parallel with eq. (2.3). However, this is clearly an inelegant and difficult approach, and standard practice is to choose *ansatze* for the vertices. A suitable *ansatz* will respect all appropriate symmetries, gauge symmetry and supersymmetry in this case, and be free of kinematic singularities.

A necessary condition for a vertex, or indeed any proper function to respect the symmetries of a theory is obedience of the relevant Ward identities. This approach was first used by Ball and Chiu [25]. They constructed the most general possible form of a vertex that would conform to the Ward-Takahashi identity (WTI), in both fermionic and scalar (non-supersymetric) QED. That is to say, they found the most general possible form for the electron-photon and the scalar-photon vertices that would satisfy the WTIs. Their reasoning holds in the supersymmetric case so that the effective photon vertices used in this paper are consistent with theirs. (Ball and Chiu did not derive the vertices for the auxiliary fields but the adaption of their derivation for the standard scalar-photon vertex is trivial.) An analogous approach was used by Iliopolous and Zumino to express the sfermion propagators in terms of the fermion one by deriving the supersymmetric Ward identities that relate them. We shall be using their results in this paper although our use of the WZ gauge will require a short discussion first.

Finally, relating the form of the photino vertices, which were unknown, to those of the photon, which were partly known, was completed by the author and a co-worker in a previous paper [17]. That work was highly analogous to the paper of Ball and Chiu [25]. Another paper by the current author rederived the same results using an independant method [26]. We outline both methods in the next two sections.

3. Finding the SWIs: The Effective Action Approach

The standard way to derive Ward identities relating the proper functions is to take functional derivatives of the effective action Γ. To do so with the identity transformation of eq. (2.2) generally requires the use of ghost fields, because of the nonlinear terms. Indeed the rigorous derivation of SUSY BRST invariance in WZ gauge has been discussed in [27]–[31] for SUSY Yang-Mills theories. However, since we are working in an abelian theory we can take advantage of the fact that the ghosts are noninteracting.

To begin our derivation, it is important to remember that eq. (2.1) is not the full theory but a supergauge projection of it. The full lagrangian of SQED, first given by Wess and Zumino [24], is invariant to the SUSY transformation

$$\begin{aligned}
\delta a &= -i\bar{\zeta}\psi\,, \\
\delta b &= \bar{\zeta}\gamma_5\psi\,, \\
\delta\psi &= [f + i\gamma_5 g + i\gamma\cdot\partial(a + i\gamma_5 b)]\zeta\,, \\
\delta f &= \bar{\zeta}[\gamma\cdot\partial\psi]\,, \\
\delta g &= i\bar{\zeta}[\gamma_5\gamma\cdot\partial\psi]\,, \\
\delta C &= \bar{\zeta}\gamma_5\chi\,, \\
\delta\chi &= [M + i\gamma_5 N + i\gamma\cdot(A - i\gamma_5\partial C)]\zeta\,, \\
\delta M &= \bar{\zeta}[\gamma\cdot\partial\chi]\,, \\
\delta N &= i\bar{\zeta}[\gamma_5\gamma\cdot\partial\chi]\,, \\
\delta A_\mu &= \bar{\zeta}\gamma_\mu\lambda\,, \\
\delta\lambda &= \sigma^{\nu\mu}\partial_\mu A_\nu\zeta + i\gamma_5 D\zeta\,, \\
\delta D &= i\bar{\zeta}\gamma_5\gamma\cdot\partial\lambda\,,
\end{aligned} \tag{3.1}$$

which is perfectly linear and the standard procedure yields an effective action

$$\Gamma(\phi_{cl}, \phi^*_{cl}, C_{cl}, \chi_{cl}, M_{cl}, N_{cl}, A^\mu_{cl}, \lambda_{cl}, D_{cl})\,,$$

where ϕ, ϕ^* are shorthand for the fields in the chiral multiplet ϕ and its conjugate ϕ^*. The effective action in the general Yang-Mills case is only invariant to the BRST identities [27]–[31] derived from gauge invariance, but in an abelian theory the ghost (super)fields decouple, restoring the original (super)gauge invariance. So, like the original exponential action found by Wess and Zumino [24], the effective action is invariant to transformations corresponding (i.e. with the fields replaced by their classical counterparts) to the SUSY transformation in eq. (3.1) and the supergauge transformation. The supergauge transformation allows us to gauge C_{cl},χ_{cl},M_{cl} and N_{cl} to zero, leaving us with an effective action $\Gamma(\phi_{cl}, \phi^*_{cl}, A^\mu_{cl}, \lambda_{cl}, D_{cl})$, invariant to modified SUSY transforms corresponding to those in eq. (2.2). (The subscript cl will repressed henceforth for the sake of clarity.)

Ward identities, based on SUSY, that constrain the proper functions, can now be found by taking functional derivatives of

$$\begin{aligned} 0 &= \delta_{WZ}\Gamma \\ &= \delta_{WZ}a\frac{\delta\Gamma}{\delta a} + \delta_{WZ}b\frac{\delta\Gamma}{\delta b} + \delta_{WZ}\psi_\alpha\frac{\delta\Gamma}{\delta\psi_\alpha} + \delta_{WZ}f\frac{\delta\Gamma}{\delta f} + \delta_{WZ}g\frac{\delta\Gamma}{\delta g} + \\ &\quad +\delta_{WZ}a^*\frac{\delta\Gamma}{\delta a^*} + \delta_{WZ}b^*\frac{\delta\Gamma}{\delta b^*} + \delta_{WZ}\bar{\psi}^\alpha\frac{\delta\Gamma}{\delta\bar{\psi}^\alpha} + \delta_{WZ}f^*\frac{\delta\Gamma}{\delta f^*} + \delta_{WZ}g^*\frac{\delta\Gamma}{\delta g^*} + \\ &\quad +\delta_{WZ}A_\mu\frac{\delta\Gamma}{\delta A_\mu} + \delta_{WZ}\lambda_\alpha\frac{\delta\Gamma}{\delta\lambda_\alpha} + \delta_{WZ}D\frac{\delta\Gamma}{\delta D}\,. \end{aligned} \tag{3.2}$$

Defining $\Gamma_{X\cdots Z} \equiv \frac{\delta^n\Gamma}{\delta X\cdots\delta Z}$, taking the functional derivative of eq. (3.2) with respect to $\bar{\psi}(x)$ and $(f(y), g(y))$ gives

$$\begin{aligned} \Gamma_{\bar{\psi}\psi} &\equiv \langle\psi\bar{\psi}\rangle^{-1} \\ &= -i\Gamma_{f^*a}(p) + i\gamma\cdot p\Gamma_{f^*f}(p) \\ &= -i\Gamma_{g^*b}(p) + i\gamma\cdot p\Gamma_{g^*g}(p)\,, \end{aligned} \tag{3.3}$$

whereas that with respect to $\bar{\psi}(x)$ and $(a(y), b(y))$ produces

$$\gamma\cdot p\Gamma_{\bar{\psi}\psi}(p) = i\Gamma_{a^*a}(p) - i\gamma\cdot p\Gamma_{a^*f}(p) = i\Gamma_{b^*b}(p) - i\gamma\cdot p\Gamma_{b^*g}(p)\,, \tag{3.4}$$

both of which are equivalent to the corresponding Ward identities found by Itzykson and Zuber in ref. [2].

It follows that the two-point proper functions, and therefore the propagators, for the chiral multiplet in SQED, even in WZ gauge, are the same as those in [2]. This result, while näively intuitive, is seemingly at odds with gauge-fixing which is not SUSY in WZ gauge. We shall deal with these issues in the next section.

It now remains to find the form of the vertices. This was done already in ref. [17] by taking third order functional derivatives of eq. (3.2). As an example, functionally deriving eq. (3.2) with respect to $\delta^3/(\delta a(y)\delta a^*(x)\delta\bar{\lambda}(z))$ yields the identity

$$\gamma_\mu\Gamma^\mu_{a^*A_\mu a}(p,q) = \Gamma_{\bar{\lambda}a^*\psi}(p,q)\gamma\cdot q + e(B(p^2) - B(q^2)) + \Gamma_{\bar{\lambda}a^*\psi}(-q,-p)\gamma\cdot p\,. \tag{3.5}$$

The full set of three-point SWIs [17] is rederived in appendix A.

4. Finding the SWIs: The Completely Invariant Approach

The transformation (2.2) leaves only the gauge invariant part of the action invariant, but in WZ-gauge the gauge-fixing term violates SUSY. Standard procedures for handling this have been known for some time [32, 33, 34]. They all involve the introduction of ghost fields. However we have recently discovered a modification to eq. (2.2) which restores SUSY invariance [26]. The trick is to recognise that the violation of SUSY by the gauge-fixing term is essentially no different from the violation of SUSY by the WZ-gauge. Like the WZ-gauge, the SUSY violation can be undone with a gauge transformation. In Lorentz gauge this is

$$
\begin{aligned}
\delta_L a &= i\bar{\zeta}\frac{\not{\partial}}{\Box}\lambda a\,,\\
\delta_L b &= i\bar{\zeta}\frac{\not{\partial}}{\Box}\lambda b\,,\\
\delta_L \psi &= i\bar{\zeta}\frac{\not{\partial}}{\Box}\lambda \psi\,,\\
\delta_L f &= i\bar{\zeta}\frac{\not{\partial}}{\Box}\lambda f\,,\\
\delta_L g &= i\bar{\zeta}\frac{\not{\partial}}{\Box}\lambda g\,,\\
\delta_L A_\mu &= \partial_\mu\bar{\zeta}\frac{\not{\partial}}{\Box}\lambda\,,\\
\delta_L \lambda &= 0\,,\\
\delta_L D &= 0\,.
\end{aligned}
\tag{4.1}
$$

Since δ_L is a gauge transformation it leaves the gauge-invariant part of the action unchanged, so the complete SUSY transform of SUSY QED in component notation is

$$
\delta_{WZL} = \delta_{WZ} + \delta_L\,. \tag{4.2}
$$

δ_L also has the nice feature of obeying the SUSY algebra, modulo the Lorentz gauge-fixing condition [26].

Before we go on to discuss three-point Ward identities, we need to discuss the SWIs relating the propagators. Notably, there is an identity relating the photon and photino propagators,

$$
\begin{aligned}
0 &= \langle \delta_{WZL}(A_\mu(x)\lambda(y))\rangle\\
&= \langle A_\mu(x)A_\beta(y)\rangle_y\partial_\alpha\sigma^{\beta\alpha}\zeta - \langle\lambda(y)\bar{\lambda}(x)\rangle\gamma_\mu\zeta + \langle\lambda(y)\bar{\lambda}(x)\rangle\frac{{}_x\partial_\mu}{{}_x\not{\partial}}\zeta\,,
\end{aligned}
\tag{4.3}
$$

After converting to momentum space, the surviving members of the gauge multiplet have the following propagators in WZ gauge:

$$
\langle A_\mu A_\nu\rangle(k) = -\frac{1}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{1+\Pi(k)} + \xi\frac{k_\mu k_\nu}{k^4}\,, \tag{4.4}
$$

$$
\langle\lambda\bar{\lambda}\rangle(k) = \frac{-i}{1+\Pi(k)\,\not{k}}\,, \tag{4.5}
$$

$$
\langle DD\rangle(k) = \frac{1}{1+\Pi(k)}\,. \tag{4.6}
$$

This relationship between the photon and photino propagators is what one would naively expect [16].

We shall also need the identities relating the electron and selectron propagators. One might expect that since their wavefunction renormalisation is dependent on the gauge parameter ξ, the SUSY violation of the gauge-fixing term would cause the electron and selectron wavefunction renormalisations to differ, at least nonperturbatively. However our transformation respects covariant gauge-fixing so such reasoning does not apply. The Ward identity relating the ψ and a propagators is

$$\begin{aligned} 0 &= \langle \delta_{WZL}(\bar{\psi}(x)a(y))\rangle \\ &= -i\bar{\zeta}\langle\psi(y)\bar{\psi}(x)\rangle + \bar{\zeta}\langle a(y)f^*(x)\rangle - i\bar{\zeta}\ {}_x\not{\partial}\langle a(y)a^*(x)\rangle\,, \end{aligned} \tag{4.7}$$

as found originally in [2]. The non-linear contribution to this and all propagator Ward identities vanishes by the cluster decomposition principle [35, 36, 37]. Taking the most general form for the electron propagator

$$\langle\psi\bar{\psi}\rangle(p) = \frac{i}{\not{p}A(p^2) + B(p^2)}\,, \tag{4.8}$$

we then have

$$\langle aa^*\rangle(p^2) = \frac{A(p^2)}{p^2A(p^2)^2 - B(p^2)^2}, \tag{4.9}$$

$$\langle af^*\rangle(p^2) = \langle fa^*\rangle(p^2) = \frac{B(p^2)}{p^2A(p^2)^2 - B(p^2)^2}\,, \tag{4.10}$$

$$\langle ff^*\rangle(p^2) = \frac{p^2A(p^2)}{p^2A(p^2)^2 - B(p^2)^2}\,, \tag{4.11}$$

where the last two propagators are given by [2]

$$\begin{aligned} 0 &= \langle \delta_{WZL}(\bar{\psi}(x)f(y))\rangle \\ &= \bar{\zeta}_y\ \not{\partial}\langle\psi(y)\bar{\psi}(x)\rangle + \bar{\zeta}\langle f(y)f^*(x)\rangle - i\bar{\zeta}_x\ \not{\partial}\langle f(y)a^*(x)\rangle\,. \end{aligned} \tag{4.12}$$

Similarly,

$$\langle bb^*\rangle(p^2) = \frac{A(p^2)}{p^2A(p^2)^2 - B(p^2)^2}, \tag{4.13}$$

$$\langle bg^*\rangle(p^2) = \langle gb^*\rangle(p^2) = \frac{B(p^2)}{p^2A(p^2)^2 - B(p^2)^2}\,, \tag{4.14}$$

$$\langle gg^*\rangle(p^2) = \frac{p^2A(p^2)}{p^2A(p^2)^2 - B(p^2)^2}\,. \tag{4.15}$$

Since δ_{WZL} is both nonlinear and nonlocal we cannot derive vertex identities the effective action or any other method requiring the Legendre transform. This problem was circumvented in [26] by first finding identities for the three point Green's functions and then taking suitable linear combinations of these to find identities relating the vertices. We shall illustrate the principle here and refer the reader to [26] for the full calculation.

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Consider

$$\begin{aligned}
0 &= \langle \delta_{WZL}(a(p)A_\mu(p-q)\bar{\psi}(q))\rangle \\
&= i\langle\bar{\psi}(q)A_\mu(p-q)\psi(p)\rangle - \left(\gamma_\mu - \frac{(p-q)_\mu}{\not p - \not q}\right)\langle\bar{\psi}(q)\lambda(p-q)a(p)\rangle \\
&\quad +\langle f^*(q)A_\mu(p-q)a(p)\rangle - \not q\langle a^*(q)A_\mu(p-q)a(p)\rangle \\
&\quad +e\langle A_\mu A_\nu\rangle(p-q)\gamma^\nu\langle aa^*\rangle(p)\,,
\end{aligned} \tag{4.16}$$

and

$$\begin{aligned}
0 &= \langle \delta_{WZL}(f(p)A_\mu(p-q)\bar{\psi}(q))\rangle \\
&= -i\not p\langle\bar{\psi}(q)A_\mu\psi(p)\rangle - ie\langle\psi\bar{\psi}\rangle(q)\gamma^\nu\langle A_\nu A_\mu\rangle(p-q) + \\
&\quad + \left(\gamma_\mu - \frac{(p-q)_\mu}{\not p - \not q}\right)\langle\bar{\psi}(q)\bar{\lambda}(p-q)f(p)\rangle + \langle f^*(q)A_\mu(p-q)f(p)\rangle + \\
&\quad - \not q\langle a^*(q)A_\mu(p-q)f(p)\rangle + e\langle A_\mu A_\nu\rangle(p-q)\gamma^\nu\langle fa^*\rangle(p)\,,
\end{aligned} \tag{4.17}$$

whose calculational details are given in [26]. Taking the linear combinations

$$A(p^2)\langle\delta(\bar{\psi}(q^2)A_\mu(p-q)f(p))\rangle - B(p^2)\langle\delta(\bar{\psi}(q^2)A_\mu(p-q)a(p))\rangle\,, \tag{4.18}$$

and

$$B(p^2)\langle\delta(\bar{\psi}(q^2)A_\mu(p-q)a(p))\rangle - p^2A(p^2)\langle\delta(\bar{\psi}(q^2)A_\mu(p-q)f(p))\rangle\,, \tag{4.19}$$

produces the SUSY vertex Ward identities

$$\begin{aligned}
&i\sigma^{\mu\nu}(p-q)_\nu\Gamma_{\bar{\lambda}f^*\psi}(p,q) \\
&= \Gamma^\mu_{\bar{\psi}A\psi}(p,q) - i\not q\Gamma^\mu_{f^*Af}(p,q) + i\Gamma^\mu_{f^*Aa}(p,q) - ie\gamma^\mu A(p^2)\,,
\end{aligned} \tag{4.20}$$

and

$$\begin{aligned}
i\sigma^{\mu\nu}(p-q)_\nu\Gamma_{\bar{\lambda}a^*\psi}(p,q) &= i\Gamma^\mu_{a^*Aa}(p,q) - i\not q\Gamma^\mu_{a^*Af}(p,q) - \\
&\quad -e\gamma^\mu\langle\psi\bar{\psi}\rangle^{-1}(q) - \not p\Gamma^\mu_{\bar{\psi}A\psi}(p,q) + ie\gamma^\mu B(p^2)\,,
\end{aligned} \tag{4.21}$$

respectively. Calculating the complete array of Green's functions' SUSY Ward identities and finding the complete set of useful linear combinations leads to the the same set of vertex identities as in the previous section. This gives us confidence that our vertices are correct.

Before moving on we address the point that section 3 uses δ_{WZ} while this one uses δ_{WZL}. This might seem inconsistent but it must be realised that δ_L is only a U(1) gauge transformation so including it in the derivation of section 3 only adds a U(1) Ward-Takahashi identity to the SUSY one.

5. The General Vertices of SQED

It is remarkable that, within the assumptions of charge conjugation invariance and that the scalars a and b interact identically with the photon, the SWIs combine with the Ward-Takahashi identities (WTIs) to limit the set of effective vertices to a given general form, in close analogy to Ball and Chiu restricting the ordinary QED vertices from the WTIs [25]. The full set of vertices is not needed here. This paper requires only the electron-photon and the electron-scalar-photino vertices. They are

$$\begin{aligned}
\Gamma_{\bar\lambda a^*\psi}(p,q) &= \frac{e}{p^2-q^2}(p^2A(p^2)-q^2A(q^2)) + \frac{e}{p^2-q^2}(B(p^2)-B(q^2))\gamma\cdot q + \\
&+\frac{1}{2}e(p^2-\gamma\cdot q\gamma\cdot p)T_{aa}(p^2,q^2,p\cdot q) + \\
&+\frac{1}{2}ep^2(q^2-\gamma\cdot p\gamma\cdot q)T_{ff}(p^2,q^2,p\cdot q) + \\
&+\frac{1}{2}e[\gamma\cdot p(p^2-q^2)-2\gamma\cdot q(p^2-p\cdot q)]T_{af}(p^2,q^2,p\cdot q)\,, \qquad (5.1)\\
\Gamma_{\bar\lambda f^*\psi}(p,q) &= \frac{-e}{p^2-q^2}(A(p^2)-A(q^2))\gamma\cdot q - \frac{e}{p^2-q^2}(B(p^2)-B(q^2)) + \\
&+\frac{1}{2}e(\gamma\cdot p-\gamma\cdot q)T_{aa}(p^2,q^2,p\cdot q) + \\
&+\frac{1}{2}e(p-q)^2T_{af}(p^2,q^2,p\cdot q) - \\
&-\frac{1}{2}e\gamma\cdot q(p^2-\gamma\cdot p\gamma\cdot q)T_{ff}(p^2,q^2,p\cdot q)\,, \qquad (5.2)
\end{aligned}$$

and

$$\begin{aligned}
\Gamma^\mu_{\bar\psi A_\mu\psi}(p,q) &= \Gamma^\mu_{BC}(p,q) + \frac{ie}{p^2-q^2}(A(p^2)-A(q^2))\left[\frac{1}{2}T_3^\mu - T_8^\mu\right] - \\
&-\frac{ie}{p^2-q^2}(B(p^2)-B(q^2))T_5^\mu + \frac{1}{2}ieT_{aa}(p^2,q^2,p\cdot q)T_3^\mu + \\
&+ieT_{af}(p^2,q^2,p\cdot q)\left[\frac{1}{2}(p-q)^2T_5^\mu - T_1^\mu\right] + \\
&+\frac{1}{2}ieT_{ff}(p^2,q^2,p\cdot q)[T_2^\mu - p\cdot qT_3^\mu - (p-q)^2T_8^\mu]\,, \qquad (5.3)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma^\mu_{BC}(p,q) &= \frac{1}{2}\frac{ie}{p^2-q^2}(\gamma\cdot p+\gamma\cdot q)(A(p^2)-A(q^2))(p+q)^\mu + \\
&+ie\frac{1}{2}(A(p^2)+A(q^2))\gamma^\mu + \frac{ie}{p^2-q^2}(B(p^2)-B(q^2))(p+q)^\mu\,, \qquad (5.4)\\
T_1^\mu &= p^\mu(q^2-p\cdot q)+q^\mu(p^2-p\cdot q)\,, \qquad (5.5)\\
T_2^\mu &= (\gamma\cdot p+\gamma\cdot q)T_1^\mu\,, \qquad (5.6)\\
T_3^\mu &= \gamma^\mu(p-q)^2-(\gamma\cdot p-\gamma\cdot q)(p-q)^\mu]\,, \qquad (5.7)\\
T_5^\mu &= \sigma^{\mu\nu}(p-q)_\nu\,, \qquad (5.8)\\
T_8^\mu &= \frac{1}{2}(\gamma\cdot p\gamma\cdot q\gamma^\mu - \gamma^\mu\gamma\cdot q\gamma\cdot p)\,, \qquad (5.9)
\end{aligned}$$

and represent the electron-a-photino, electron-f-photino and electron-photon vertices respectively. The scalar functions $T_{aa}(p^2, q^2, p\cdot q)$,$T_{af}(p^2, q^2, p\cdot q)$, $T_{ff}(p^2, q^2, p\cdot q)$ are symmetric in p, q due to charge conjugation invariance [17] but are unconstrained by either the Ward-Takahashi or SUSY Ward identities. In a slight abuse of language we refer to these functions as 'transverse' functions since they contribute only to the transverse components of the vertices.

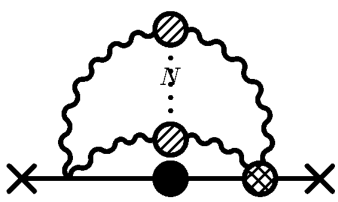
It is a trivial exercise to check that these vertices reduce to the bare ones when $A(p^2)$ and $B(p^2)$ are set to their bare level values, 1 and m, respectively. It is also easy to see that the photon vertices are consistent with the general form for photon vertices in non-SUSY QED [25].

6. The nonrenormalisation theorem in SQED

Before presenting our own analysis, we give a brief overview of the derivation of Clark and Love's nonrenormalisation theorem [4]. Since they perform their analysis in the superfield notation their action is

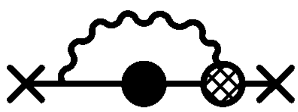
$$2+\int\int d^dx\, d^2\theta\, d^2\bar{\theta}\, \bar{\phi}\, e^{2eV} \phi\, (D^3 \bar{D}\bar{D}D\, m \frac{1}{g} D\bar{D}D\bar{D}\bar{\theta})V\phi + m \int d^dx d^2\bar{\theta}\bar{\phi}\bar{\phi}\,, \tag{6.1}$$

where the interaction terms are seen to be exponential in the vector superfield V and generate an infinite number of interactions between it and the chiral superfield. The appropriate SDE is therefore represented graphically by

$B(p^2) = m +$ 

(6.2)

This is an intractible mess and in order to make any headway some simplification is clearly needed. The approach of ref. [4] is to truncate the seagull and higher order contributions, leaving

$B(p^2) = m +$ 

(6.3)

which is obviously a lot simpler. After some algebra, the effective mass is seen to be

$$B(p^2) = m - (1 - \frac{1}{\xi})\frac{(2e)^2}{i}\int \frac{d^4q}{(2\pi)^4}\frac{p^2 B(q^2)}{A^2(q^2)q^2 + B^2(q^2)}\frac{4}{(p-q)^4}\,. \tag{6.4}$$

and the vanishing prefactor discussed earlier is demonstrated. Superficially this expression vanishes in the chiral limit in Feynman gauge, which is the basis of the nonrenormalisation theorem.

7. Chiral symmetry breaking is permitted in SQED

With the form of the effective propagators and vertices established we are now in a position to study the DSE. Performing the Wick rotation into euclidean space and substituting the full vertices into the DSE gives us the following integral equations:

$$\begin{aligned}
B(p^2) = {} & 2e^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p-q)^2}\frac{1}{p^2-q^2}[D_{af}(q^2)p^2A(p^2)+(p^2-2q^2)D_{aa}(q^2)B(p^2)]+ \\
& +(\xi-1)e^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p-q)^4}[D_{af}(q^2)p^2A(p^2)+D_{aa}(q^2)q^2B(p^2)]- \\
& -(\xi-1)e^2\int\frac{d^dq}{(2\pi)^d}\frac{p\cdot q}{(p-q)^4}[D_{af}(q^2)A(p^2)+D_{aa}(q^2)B(p^2)]- \\
& -\frac{1}{2}e^2\int\frac{d^dq}{(2\pi)^d}D_{af}(q^2)T_{aa}(p^2,q^2,p\cdot q)- \\
& -e^2\int\frac{d^dq}{(2\pi)^d}D_{aa}(q^2)T_{af}(p^2,q^2,p\cdot q)\left[\frac{(p\cdot q)^2-p^2q^2}{(p-q)^2}+q^2-p\cdot q\right]+ \\
& +\frac{1}{2}e^2\int\frac{d^dq}{(2\pi)^d}D_{af}(q^2)T_{ff}(p^2,q^2,p\cdot q)\left[p^2\frac{q^2-p\cdot q}{(p-q)^2}+q^2\frac{p^2-p\cdot q}{(p-q)^2}\right],
\end{aligned} \tag{7.1}$$

$$\begin{aligned}
A(p^2)-1 = {} & 2e^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p-q)^2}\frac{1}{p^2-q^2}D_{aa}(q^2)\left[(p^2-2q^2)A(p^2)+q^2A(q^2)\right]- \\
& -2e^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p-q)^2}\frac{1}{p^2-q^2}D_{af}(q^2)[B(p^2)-B(q^2)]+ \\
& +(\xi-1)e^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p-q)^4}D_{aa}(q^2)q^2[A(p^2)+A(q^2)]- \\
& -(\xi-1)e^2\int\frac{d^dq}{(2\pi)^d}\frac{1}{(p-q)^4}D_{af}(q^2)[B(p^2)-B(q^2)]+ \\
& +(\xi-1)\frac{e^2}{p^2}\int\frac{d^dq}{(2\pi)^d}\frac{p\cdot q}{(p-q)^4}D_{af}(q^2)[B(p^2)-B(q^2)]- \\
& -(\xi-1)\frac{e^2}{p^2}\int\frac{d^dq}{(2\pi)^d}\frac{p\cdot q}{(p-q)^4}D_{aa}(q^2)[p^2A(p^2)+q^2A(q^2)]+ \\
& +\frac{e^2}{p^2}\int\frac{d^dq}{(2\pi)^d}D_{aa}(q^2)T_{aa}(p^2,q^2,p\cdot q)\left[\frac{3}{2}p\cdot q-p^2\frac{q^2-p\cdot q}{(p-q)^2}-p\cdot q\frac{p^2-p\cdot q}{(p-q)^2}\right]+ \\
& +\frac{1}{2}\frac{e^2}{p^2}\int\frac{d^dq}{(2\pi)^d}D_{af}(q^2)T_{af}(p^2,q^2,p\cdot q)\left[p\cdot q-3p^2+2(p^2-p\cdot q)\frac{q^2-p\cdot q}{(p-q)^2}\right]+ \\
& +\frac{3}{2}\frac{e^2}{p^2}\int\frac{d^dq}{(2\pi)^d}D_{aa}(q^2)T_{ff}(p^2,q^2,p\cdot q)q^2p^2,
\end{aligned} \tag{7.2}$$

The contribution to $B(p^2)$ from the transverse components of the vertices is referred to as the transverse contribution. Eqs. (7.1), (7.2) were derived in the quenched approximation. The effects of vacuum polarisation will be considered shortly.

Considering first the minimal SUSY Ball-Chiu *ansatz* where the transverse functions are set to zero, we see immediately that there is no reason why $B(p^2)$ must vanish. We can also eliminate the possibility of the transverse functions inducing a nonrenormalisation theorem by cancelling off the minimal contribution. To see this, recall that the transverse functions are symmetric in p and q and examine the coefficients of $D_{af}(q^2)$. Its coefficients in the minimal contribution are asymmetric in p, q because of the $A(p^2)$ factor whereas those in the transverse contribution are exactly symmetric. (A corresponding argument for terms in eq. (7.1) proportional to $D_{aa}(q^2)$ cannot be made because the transverse contribution has an asymmetric component.) It follows that the integrand of $B(p^2)$ will not vanish, regardless of the choice of transverse functions. This result will still hold after angular integration as the transverse contribution will be a symmetric function multiplied by q^{d-1} whereas the minimal contribution will not. However we cannot eliminate the possibility that eqs. (7.1), (7.2) may simply not be solvable unless $B(p^2)$ is set to zero.

This apparent contradiction between ourselves and the superfield analyses [4, 6] requires explanation. If an achiral solution is forbidden in the superfield formalism then it must also be forbidden in the component formalism, and yet our analysis finds no evidence that an achiral solution cannot exist. We can eliminate the possibility that our choice of the quenched approximation has obscured a nonrenormalisation theorem. Indeed, using the propagators given in eqs. (4.4) to (4.6), we see that he right hand sides of eqs. (7.1), (7.2) are simply multiplied by a factor of $\frac{1}{1+\Pi(p^2)}$ in Landau gauge ($\xi = 0$), which preserves our result. (The choice of a specific gauge for our calculation is permitted by the absence of approximations.)

The derivation of the superfield DSE by both Clark *et. al.* and Campbell-Smith *et. al.* , is perfectly valid within the approximations used. These approximations, as stated by Clark and Love [4], consist of choosing the three-point vertex to satisfy the differential U(1) Ward identity instead of the full Ward-Takahashi identity, and truncating all terms with seagull and higher n-point vertices from the DSE. This latter approximation, as ref. [4] points out, explicitly violates supergauge covariance, even if the three-point vertex is covariantly dressed.

This seems to support Kaiser and Selipsky's [5] assertion that it is not valid to use the gauge dependent prefactor to eliminate the dynamically generated mass. As pointed out by Kaiser *et. al.* , this prefactor might be cancelled in the limit of the Feynman gauge by some factor in the integral. Campbell-Smith *et. al.* sought to defuse this argument in $2 + 1$ dimensions by demonstrating that this did not occur. The weakness in their argument however is the set of truncations used in the derivation of their equations. They repeat Clark *et. al.* 's elimination of higher order graphs to the DSE, which violates U(1) gauge covariance. They also make use of a compactification argument requiring that one spatial dimension be compactified. All contributions not in the momentum range lower than that derived from compactification are neglected. Further simplification was achieved by assuming a trivial dressing for the vertices, thus reducing U(1) gauge covariance even further. The results of this analysis were intended only for $SQED_3$ and, as [6] points out, do not apply to $SQED_4$.

The approximations used in [6] were generally chosen so as to have minimal impact in the infrared region where chiral symmetry breaking is largely determined, but to combine U(1) gauge violating approximations with a gauge dependent argument is dangerous. Con-

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sider, for example, the equivalent of eq. (7.1) in non-SUSY QED_3 in the quenched rainbow approximation,

$$B(p^2) = (\xi + 2)\frac{e^2}{4\pi^2 p}\int_0^\infty dq \frac{qB(q^2)}{q^2A^2(q^2) + B^2(q^2)}(\ln|p+q| - \ln|p-q|)\,. \tag{7.3}$$

In the special gauge of $\xi = -2$ the right hand side of eq. (7.3) vanishes unless $A(q^2)$ and $B(q^2)$ conspire to cancel this prefactor. It does not follow though that chiral symmetry is unbroken. In fact non-SUSY QED_3 is known to break chiral symmetry from lattice studies [38, 39]. The vanishing prefactor in eq. (7.3) is an artifact of the rainbow approximation and illustrates the weakness of gauge dependent arguments.

In contrast to the two superfield analyses described above, our derivation truncates no terms from the DSE. Because it uses the most general possible form of the vertices consistent with both the SUSY Ward identities and the Ward-Takahashi identities, our argument is fully SUSY and U(1) gauge invariant. This statement must be tempered with a reminder that the calculations were done in WZ gauge which alters the form of the SWIs but any argument claiming that the WZ gauge might obscure a nonrenormalisation theorem must be false, since a result which holds in arbitrary gauge must also hold in any specific one.

8. The CJT effective potential in the Nonperturbative Regime

Finding the CJT effective potential for any theory obviously requires the correct form of the propagators. The correct form of the selectron propagators in SUSY theories is obscured by the auxiliary fields f, g, but has been known at least since the work of Miller [40] and Fogleman and Viswanathan [41]. In this paper we use our notation from ref. [17] and write the selectrons a, f (b, g) as a column matrix $[a] \equiv \binom{a}{f}$ $\left([b] \equiv \binom{b}{g}\right)$. The selectron propagator becomes

$$[D(p^2)] \equiv \begin{bmatrix} D_{aa}(p^2) & D_{af}(p^2) \\ D_{fa}(p^2) & D_{ff}(p^2) \end{bmatrix} = \begin{bmatrix} D_{bb}(p^2) & D_{bg}(p^2) \\ D_{gb}(p^2) & D_{gg}(p^2) \end{bmatrix}. \tag{8.1}$$

The CJT effective potential is

$$\begin{aligned} V[S, [D]_a, [D]_b] = &\int \frac{d^dp}{(2\pi)^d}(\mathrm{Tr}\ln(S_0^{-1}(p)S(p)) + \frac{1}{2}\mathrm{Tr}(1 - S_0^{-1}(p)S(p))) - \\ &-2\int \frac{d^dp}{(2\pi)^d}(\mathrm{Tr}\ln([D(p^2)]_0^{-1}[D(p^2)]) + \frac{1}{2}\mathrm{Tr}(1 - [D(p^2)]_0^{-1}[D(p^2)])) + \\ &+\frac{1}{2}\int \frac{d^dp}{(2\pi)^d}(\mathrm{Tr}\ln(S_{0\lambda}^{-1}(p)S_\lambda(p)) + \mathrm{Tr}(1 - S_{0\lambda}^{-1}(p)S_\lambda(p))) \\ &-\frac{1}{2}\int \frac{d^dp}{(2\pi)^d}(\mathrm{Tr}\ln(D_{\mu\nu 0}^{-1}(p)D_{\nu\sigma}(p)) + 4 - D_{\mu\nu 0}^{-1}(p)D_{\nu\mu}(p)) \\ &-\frac{1}{2}\int \frac{d^dp}{(2\pi)^d}(\mathrm{Tr}\ln(D_{0D}^{-1}(p^2)D_D(p^2)) + \mathrm{Tr}(1 - D_{0D}^{-1}(p)D_D(p^2)))\,. \end{aligned} \tag{8.2}$$

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(The calculation in [23] used ordinary scalar propagators $D_{aa}(p^2)$.) Substituting eqs. (4.8) to (4.15) and eqs. (4.4) to (4.6) into eq. (8.2) reveals that the CJT effective potential is zero at its extrema, and therefore uniformly zero. Note that there is another term in eq. (8.2), given by

$$\int \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3} D_{\mu\nu}(q)\,\mathrm{Tr}([\Gamma^{\mu\nu}_{a^* A_\mu A_\nu a}(p,p,q)][D(p^2)])\,, \tag{8.3}$$

where $[\Gamma^{\mu\nu}_{a^* A_\mu A_\nu a}]$ is the four-point vertex for two $[a]$s and two photons, corresponding to the vacuum graphs that give rise to the tadpole contributions in the boson DSE. As we have written it, eq. (8.2) only gives half of these particular diagrams. However, eq. (8.3) is proportional to the massless tadpole integral and is therefore zero by dimensional regularisation [42].

So Pisarski's result is not restricted to the many flavour limit, i.e. the nonrenormalisation theorem for the effective potential also applies nonperturbatively. It is trivial to extend our result to arbitrary numbers of flavours and it follows that the CJT effective potential cannot be used to select the dynamically favoured propagator in a SUSY theory without SUSY breaking.

9. Conclusion

There is no reason to suppose the nonexistence of an achiral solution to the DSE in either SQED_3 or SQED_4, and no nonperturbative nonrenormalisation theorem in either of these theories. The apparent theorem found in superfield treatments is an artifact of the substantial approximations required by the form of the DSE in the superfield notation. It must be admitted, however, that an actual achiral solution, numerical or algebraic, has so far eluded us, so we have yet to prove that such a solution actually exists. The question of chiral symmetry breaking in SQED will almost certainly remain open until such a solution is either found or, by some other means, proved nonexistent.

If, as our form of the DSE suggests, both solutions do exist, then we have to address the issue of whether it is the chiral or the achiral solution that is dynamically favoured. We have demonstrated, however, that the CJT effective potential cannot be used to determine this in the absence of SUSY breaking since it is uniformly zero, even in the nonperturbative regime.

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Functional Derivative of $\delta_{WZ}\Gamma=0$	SWI	Functional Derivative of $\delta_{WZ}\Gamma=0$	SWI
$\delta^3\Gamma/(\delta a(y)\delta a^*(x)\delta\bar{\lambda}(z))$	A.1	$\delta^3\Gamma/(\delta\psi(y)\delta D(z)\delta a^*(x))$	A.15
$\delta^3\Gamma/(\delta b(y)\delta b^*(x)\delta\bar{\lambda}(z))$	A.2	$\delta^3\Gamma/(\delta\psi(y)\delta D(z)\delta b^*(x))$	A.16
$\delta^3\Gamma/(\delta f(y)\delta a^*(x)\delta\bar{\lambda}(z))$	A.3	$\delta^3\Gamma/(\delta\psi(y)\delta D(z)\delta f^*(x))$	A.17
$\delta^3\Gamma/(\delta g(y)\delta b^*(x)\delta\bar{\lambda}(z))$	A.4	$\delta^3\Gamma/(\delta\psi(y)\delta D(z)\delta g^*(x))$	A.18
$\delta^3\Gamma/(\delta f(y)\delta f^*(x)\delta\bar{\lambda}(z))$	A.5	$\delta^3\Gamma/(\delta b(y)\delta D(z)\delta a^*(x))$	A.19
$\delta^3\Gamma/(\delta g(y)\delta g^*(x)\delta\bar{\lambda}(z))$	A.6	$\delta^3\Gamma/(\delta a(y)\delta\lambda(z)\delta b^*(x))$	A.20
$\delta^3\Gamma/(\delta\psi(y)\delta A_\mu(z)\delta f^*(x))$	A.10	$\delta^3\Gamma/(\delta g(y)\delta\lambda(z)\delta a^*(x))$	A.21
$\delta^3\Gamma/(\delta\psi(y)\delta A_\mu(z)\delta g^*(x))$	A.11	$\delta^3\Gamma/(\delta f(y)\delta\lambda(z)\delta b^*(x))$	A.22
$\delta^3\Gamma/(\delta\psi(y)\delta A_\mu(z)\delta a^*(x))$	A.7	$\delta^3\Gamma(\delta a(y)\delta\lambda(z)\delta g^*(x))$	A.23
$\delta^3\Gamma/(\delta\psi(y)\delta A_\mu(z)\delta b^*(x))$	A.8	$\delta^3\Gamma/(\delta b(y)\delta\lambda(z)\delta f^*(x))$	A.24
$\delta^3\Gamma/(\delta\psi_\alpha(y)\delta\bar{\psi}^\beta(x)\delta\lambda_\kappa(z))$	A.12	$\delta^3\Gamma/(\delta g(y)\delta\lambda(z)\delta f^*(x))$	A.25
		$\delta^3\Gamma/(\delta f(y)\delta\lambda(z)\delta g^*(x))$	A.26

Table 1: Each SWI is derived from a functional derivative of $\delta_{WZ}\Gamma=0$. The functional derivative leading to each SWI (indicated by its equation number) is given in this table.

A. Derivation of the Supersymmetric Ward Identities

The SWIs used to find the vertices used in this paper are based on the effective action of SQED being invariant to the SUSY transform given by δ_{WZ} in eq. (2.2), or the bare action under As already mentioned, the derivation of the SWIs used in this paper was given in ref. [17]. We repeat the derivation here for the sake of completeness.

The identities are found by taking functional derivatives of $\delta_{WZ}\Gamma=0$ where Γ is the effective action and δ_{WZ} is defined in eq. (2.2). The functional derivatives of $\delta_{WZ}\Gamma=0$ corresponding to the following SWIs are given in table 1:

$$\begin{aligned}
\gamma_\mu\Gamma^\mu_{a^*A_\mu a}(p,q) &= \Gamma_{\bar{\lambda}a^*\psi}(p,q)\gamma\cdot q + e(B(p^2)-B(q^2)) + \\
&\quad +\Gamma_{\bar{\lambda}a^*\psi}(-q,-p)\gamma\cdot p\,, && \text{(A.1)}\\
\gamma_\mu\Gamma^\mu_{b^*A_\mu b}(p,q) &= -i\Gamma_{\bar{\lambda}b^*\psi}(p,q)\gamma_5\gamma\cdot q - e(B(p^2)-B(q^2)) - \\
&\quad -i\Gamma_{\bar{\lambda}b^*\psi}(-q,-p)\gamma_5\gamma\cdot p\,, && \text{(A.2)}\\
\gamma_\mu\Gamma^\mu_{f^*A_\mu a}(p,q) + eA(p^2) &= \Gamma_{\bar{\lambda}a^*\psi}(-q,-p) + \Gamma_{\bar{\lambda}f^*\psi}(p,q)\gamma\cdot q\,, && \text{(A.3)}\\
\gamma_\mu\Gamma^\mu_{g^*A_\mu b}(p,q) - eA(p^2) &= i\Gamma_{\bar{\lambda}b^*\psi}(-q,-p)\gamma_5 + i\Gamma_{\bar{\lambda}g^*\psi}(p,q)\gamma\cdot q\gamma_5\,, && \text{(A.4)}\\
\gamma_\mu\Gamma^\mu_{f^*A_\mu f}(p,q) &= \Gamma_{\bar{\lambda}f^*\psi}(-q,-p) - \Gamma_{\bar{\lambda}f^*\psi}(p,q)\,, && \text{(A.5)}\\
\gamma_\mu\Gamma^\mu_{g^*A_\mu g}(p,q) &= i\Gamma_{\bar{\lambda}g^*\psi}(-q,-p)\gamma_5 - i\Gamma_{\bar{\lambda}g^*\psi}(p,q)\gamma_5\,, && \text{(A.6)}\\
i\sigma^{\mu\nu}(p-q)_\nu\Gamma_{\bar{\lambda}a^*\psi}(p,q) &= i\Gamma^\mu_{a^*A_\mu a}(p,q) - i\gamma\cdot q\Gamma^\mu_{a^*A_\mu f}(p,q) - e\gamma^\mu S^{-1}(p) - \\
&\quad -\gamma\cdot p\Gamma^\mu_{\bar{\psi}A_\mu\psi}(p,q) + ie\gamma^\mu B(p^2)\,, && \text{(A.7)}
\end{aligned}$$

$$i\sigma^{\mu\nu}(p-q)_\nu\Gamma_{\bar\lambda b^*\psi}(p,q) = -\gamma_5\Gamma^\mu_{b^*A_\mu b}(p,q)+\gamma_5\gamma\cdot q\Gamma^\mu_{b^*A_\mu g}(p,q) - i\gamma_5 e\gamma^\mu S^{-1}(p) - \tag{A.8}$$

$$-i\gamma_5\gamma\cdot p\Gamma^\mu_{\bar\psi A_\mu\psi}(p,q) - e\gamma_5\gamma^\mu B(p^2)\,. \tag{A.9}$$

$$\begin{aligned} i\sigma^{\mu\nu}(p-q)_\nu\Gamma_{\bar\lambda f^*\psi}(p,q) &= \Gamma^\mu_{\bar\psi A_\mu\psi}(p,q) - i\gamma\cdot q\Gamma^\mu_{f^*A_\mu f}(p,q) + \\ &\quad +i\Gamma^\mu_{f^*A_\mu a}(p,q) - ie\gamma^\mu A(p^2)\,, \end{aligned} \tag{A.10}$$

$$\begin{aligned} i\sigma^{\mu\nu}(p-q)_\nu\Gamma_{\bar\lambda g^*\psi}(p,q) &= i\gamma_5\Gamma^\mu_{\bar\psi A_\mu\psi}(p,q) + \gamma_5\gamma\cdot q\Gamma^\mu_{g^*A_\mu g}(p,q) - \\ &\quad -\gamma_5\Gamma^\mu_{g^*A_\mu b}(p,q) + e\gamma_5\gamma^\mu A(p^2)\,, \end{aligned} \tag{A.11}$$

From

$$\begin{aligned} 0 &= -i(\gamma\cdot q)_\sigma{}^\alpha(\Gamma_{\bar\psi f\lambda}(p,q))_\beta{}^\kappa + (\gamma_5\gamma\cdot q)_\sigma{}^\alpha(\Gamma_{\bar\psi g\lambda}(p,q))_\beta{}^\kappa - \\ &\quad -i(\gamma\cdot pC)_{\beta\sigma}\frac{\delta^2}{\delta\psi_\alpha(q)\delta f^*(p)}\left(\frac{\delta\Gamma}{\delta\lambda(p-q)}C^{-1}\right)^\kappa - \\ &\quad -(\gamma_5\gamma\cdot pC)_{\beta\sigma}\frac{\delta^2}{\delta\psi_\alpha(q)\delta g^*(p)}\left(\frac{\delta\Gamma}{\delta\lambda(p-q)}C^{-1}\right)^\kappa - \\ &\quad -(\gamma_5(\gamma\cdot p-\gamma\cdot q))_\sigma{}^\kappa(\Gamma_{\bar\psi D\psi}(p,q))_\beta{}^\alpha, \end{aligned} \tag{A.12}$$

where C is the charge conjugation matrix, we obtain

$$\begin{aligned} 0 &= (\gamma\cdot p-\gamma\cdot q)\gamma_5\,\mathrm{Tr}(\Gamma_{\bar\psi D\psi}(p,q)) + \gamma_\mu\,\mathrm{Tr}(\Gamma^\mu_{\bar\psi A_\mu\psi}(p,q)) + i\Gamma_{\bar\psi a\lambda}(p,q) - \\ &\quad -\gamma_5\Gamma_{\bar\psi b\lambda}(p,q) - i\Gamma_{\bar\psi a\lambda\psi}(-q,-p) + \gamma_5\Gamma_{\bar\psi b\lambda}(-q,-p) - \\ &\quad -i\gamma\cdot q\Gamma_{\bar\psi f\lambda}(p,q) + \gamma_5\gamma\cdot q\Gamma_{\bar\psi g\lambda}(p,q) - i\gamma\cdot p\Gamma_{\bar\psi f\lambda}(-q,-p) + \\ &\quad +\gamma_5\gamma\cdot p\Gamma_{\bar\psi g\lambda}(-q,-p)\,, \end{aligned} \tag{A.13}$$

by setting $\beta=\alpha$ and summing, and

$$\begin{aligned} 0 &= i\,\mathrm{Tr}(\Gamma_{\bar\psi a\lambda}(p,q)) - \gamma_5\,\mathrm{Tr}(\Gamma_{\bar\psi b\lambda}(p,q)) - i\gamma\cdot q\,\mathrm{Tr}(\Gamma_{\bar\psi f\lambda}(p,q)) + \\ &\quad +\gamma_5\gamma\cdot q\,\mathrm{Tr}(\Gamma_{\bar\psi g\lambda}(p,q)) - i\Gamma_{\bar\lambda a^*\psi}(p,q) + \gamma_5\Gamma_{\bar\lambda b^*\psi}(p,q) - \\ &\quad -i\gamma\cdot p\Gamma_{\bar\lambda f^*\psi}(p,q) - \gamma\cdot p\gamma_5\Gamma_{\bar\lambda g^*\psi}(p,q)) + \gamma_\mu\Gamma^\mu_{\bar\psi A_\mu\psi}(p,q) - \\ &\quad -\gamma_5(\gamma\cdot p-\gamma\cdot q)\Gamma_{\bar\psi D\psi}(p,q)\,, \end{aligned} \tag{A.14}$$

by setting $\beta=\kappa$ and summing.

Finally there are the SWIs governing the vertices of the D particle;

$$i\gamma_5\Gamma_{\bar\lambda a^*\psi}(p,q) = \gamma\cdot p\Gamma_{\bar\psi D\psi}(p,q) + \gamma_5\Gamma_{a^*Db}(p,q) - \gamma_5\gamma\cdot q\Gamma_{a^*Dg}(p,q)\,, \tag{A.15}$$

$$i\gamma_5\Gamma_{\bar\lambda b^*\psi}(p,q) = i\gamma_5\gamma\cdot p\Gamma_{\bar\psi D\psi}(p,q) - i\Gamma_{b^*Da}(p,q) + i\gamma\cdot q\Gamma_{b^*Df}(p,q)\,, \tag{A.16}$$

$$\gamma_5\Gamma_{f^*Db}(p,q) = i\gamma_5\Gamma_{\bar\lambda f^*\psi}(p,q) + \gamma_5\gamma\cdot q\Gamma_{f^*Dg}(p,q) + \Gamma_{\bar\psi D\psi}(p,q)\,, \tag{A.17}$$

$$\gamma_5\Gamma_{g^*Da}(p,q) = -\Gamma_{\bar\lambda g^*\psi}(p,q) + \gamma_5\gamma\cdot q\Gamma_{g^*Df}(p,q) - \Gamma_{\bar\psi D\psi}(p,q), \tag{A.18}$$

$$\begin{aligned} \gamma_5(\gamma\cdot p-\gamma\cdot q)\Gamma_{a^*Db}(p,q) &= \Gamma_{\bar\lambda b^*\psi}(-q,-p)\gamma\cdot p + i\Gamma_{\bar\lambda a^*\psi}(p,q)\gamma\cdot q\gamma_5 + \\ &\quad +ie\gamma_5(B(p^2)-B(q^2))\,, \end{aligned} \tag{A.19}$$

$$\begin{aligned} \gamma_5(\gamma\cdot p-\gamma\cdot q)\Gamma_{b^*Da}(p,q) &= i\Gamma_{\bar\lambda a^*\psi}(-q,-p)\gamma\cdot p\gamma_5 + \Gamma_{\bar\lambda b^*\psi}(p,q)\gamma\cdot q + \\ &\quad +ie\gamma_5(B(p^2)-B(q^2))\,, \end{aligned} \tag{A.20}$$

$$\gamma_5(\gamma\cdot p-\gamma\cdot q)\Gamma_{a^*Dg}(p,q) = \Gamma_{\bar\lambda g^*\psi}(-q,-p)\gamma\cdot p - i\Gamma_{\bar\lambda a^*\psi}(p,q)\gamma_5 + ie\gamma_5 A(q^2)\,, \tag{A.21}$$
$$\gamma_5(\gamma\cdot p-\gamma\cdot q)\Gamma_{b^*Df}(p,q) = i\Gamma_{\bar\lambda f^*\psi}(-q,-p)\gamma\cdot p\gamma_5 - \Gamma_{\bar\lambda b^*\psi}(p,q) + ie\gamma_5 A(q^2)\,, \tag{A.22}$$
$$\gamma_5(\gamma\cdot p-\gamma\cdot q)\Gamma_{g^*Da}(p,q) = \Gamma_{\bar\lambda g^*\psi}(p,q)\gamma\cdot q + i\Gamma_{\bar\lambda a^*\psi}(-q,-p)\gamma_5 - ie\gamma_5 A(p^2)\,, \tag{A.23}$$
$$\gamma_5(\gamma\cdot p-\gamma\cdot q)\Gamma_{f^*Db}(p,q) = i\Gamma_{\bar\lambda f^*\psi}(p,q)\gamma\cdot q\gamma_5 + \Gamma_{\bar\lambda b^*\psi}(-q,-p) - ie\gamma_5 A(p^2)\,, \tag{A.24}$$
$$\gamma_5(\gamma\cdot p-\gamma\cdot q)\Gamma_{f^*Dg}(p,q) = \Gamma_{\bar\lambda g^*\psi}(-q,-p) - i\Gamma_{\bar\lambda f^*\psi}(p,q)\gamma_5\,, \tag{A.25}$$
$$\gamma_5(\gamma\cdot p-\gamma\cdot q)\Gamma_{g^*Df}(p,q) = i\Gamma_{\bar\lambda f^*\psi}(-q,-p)\gamma_5 - \Gamma_{\bar\lambda g^*\psi}(p,q)\,. \tag{A.26}$$

These make up the entire set of SWIs containing only three-or-fewer point proper functions, modulo charge conjugation. A suitable vertex ansatz must also be consistent with the WTIs;

$$(p-q)_\mu[\Gamma_{(a.b)^*A_\mu(a,b)}]^\mu(p,q) = e[\Gamma_{(a,b)^*(a,b)}](p) - e[\Gamma_{(a,b)^*(a,b)}](q)\,, \tag{A.27}$$
$$(p-q)_\mu\Gamma^\mu_{\bar\psi A_\mu\psi}(p,q) = eS^{-1}(p) - eS^{-1}(q)\,. \tag{A.28}$$

We also have from charge conjugation invariance that

$$\begin{aligned}[\Gamma_{\bar\psi(a,b)\lambda}](p,q) &= -C[\Gamma_{\bar\lambda(a^*,b^*)\psi}](-q,-p)^TC^{-1}\,,\\ [\Gamma_{(a^*,b^*)D(b,a)}](p,q) &= -[\Gamma_{(b^*,a^*)D(a,b)}](-q,-p)\,.\end{aligned} \tag{A.29}$$

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